AA7-1 investigation Name ______________________________________

Working with i

In this investigation we are going to introduce the imaginary number, i, and learn five ways in which we work with i as a number. In the next few classes we will see how we use i in order to find complex roots of polynomials.

x	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12
x^2																	

1. Square roots of negative numbers To simplify square roots we use the property $\sqrt{ab} = (\sqrt{a})(\sqrt{b})$, using perfect squares as factors. For example:

$$\sqrt{32} = \sqrt{(16)(2)} = (\sqrt{16})(\sqrt{2}) = 4\sqrt{2}$$

Simplify the following square roots.

a. $\sqrt{27}$ b. $\sqrt{48}$ c. $\sqrt{18}$ d. $\sqrt{150}$

As you know, we cannot take the square root of a negative number. For this reason, mathematicians created an imaginary number, i.

Definition: i is a **complex number** such that i^2 = - 1. We often think of it as $\sqrt{-1}$.

i allows us to rewrite and work with the square root of negative numbers.

For example:

$$\sqrt{-9} = (\sqrt{9})(\sqrt{-1}) = 3i$$

Simplify the following square roots.

a. $\sqrt{-25}$ = b. $\sqrt{-36}$ = c. $10\sqrt{-16}$ = d. $\sqrt{-144}$

2. Arithmetic with *i*: In most ways, working with i is the same as working with a variable like x. Simplify these expressions. (If you think it is too easy you are doing it right.)

a. $2i + 4i$ = b. $5(3i)$ = c. $-4(2i)$ = d. $3i - 9i$ =

e. (2 + 3i) + (8 + 4i) = f. (5 + 7i) – (9 + 5i) =

3. This changes when we are **working with powers of** i. While x^2 can't be simplified, i^2 can and must be. $\boxed{\text{Remember } i^2 = -1.}$ Using this definition, simplify the following expressions:

a. $(2i)(5i) =$ b. $(3i)^2 =$ c. $(-6i)(4i) =$ d. $(5i^2)(-4i)^2 =$

4. Working with Complex numbers: All complex numbers have the form **a + *bi***, the sum of a real number and an imaginary number. For example, $3i = 0 + 3i$.

Next class we will find the complex roots of quadratic equations. Complex roots always come in pairs called **complex conjugates**: a pair of complex numbers in the form: (a + bi) and (a – bi).

Multiply the complex conjugates below like you would the factors of a quadratic equation and simplify each expression.

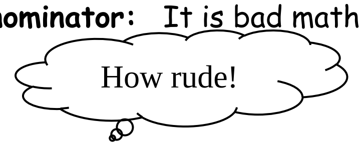
a. (4 + i)(4 - i) =

b. (5 – 3i)(5 + 3i) =

What did you notice? **The product of complex conjugates is:** $(a+bi)(a-bi) =$ ______________.

5. Rationalizing the denominator: It is bad math manners to write a fraction with a root in the denominator:

$$\frac{1}{\sqrt{3}}$$



In order to create a "more polite" fraction, we use a process called **rationalizing the denominator**.

For example: $\frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{\sqrt{3}}{\sqrt{3}^2} = \frac{\sqrt{3}}{3}$

Multiply by 1

We also don't leave *i* in the denominator. Use the same process to rationalize the denominator.

$$\frac{1}{3i}\left(\frac{\square}{\square}\right) = \frac{\quad}{\square} = \frac{\quad}{\square}$$

Multiply by 1